

Space-time encoding

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November 27, 2001

Abstract

It is widely believed that one signal can carry maximum one bit of information, and minimum $KT\log 2$ energy is required to transmit or to erase a bit. Here it is shown four bits of information can be transmitted by one signal in four dimensional space-time. It implies space-time cannot be ignored to determine the minimal energy cost of information processing.

Before the turn of the 19th century, Bell's invention of telephone and J. C. Bose's first demonstration of radiowave transmission heralded a new era of information transfer. In the following years, although these new techniques of almost instantaneous information transmission were rapidly developed and widely implemented, but it was not known how to quantify information. In 1940s, Shannon [1] proposed entropy as a measure of information, which has been accepted by all. So, information is not an abstract thing, it can be expressed in terms of entropy which is basically a thermodynamical quantity. Moreover, information is carried by electromagnetic signal. Due to these reasons many authors conjectured that the laws of physics would determine the fundamental issues of information processing. In favor of this belief, one can say information cannot be transmitted faster than the velocity of light. But for other issues, the connection between physics and information theory is yet to be understood. As for example, we don't know how to incorporate space-time with information processing. In the existing encoding space-time has no explicit role. As information is processed within the space-time, one can look for a direct connection between the two. In this paper we shall try to establish such connection.

In information theory, the basic unit of information is called bit, which is represented by an electromagnetic signal. Now, the question is : How many bits can be transmitted by a signal ? The general wisdom is that not more than one bit can be transmitted by a signal, however there is no fundamental reason why it would be so.

Let us judge a signal according to special theory of relativity. According to special theory of relativity, an event needs four dimensions for its complete specification - of which three are space dimensions and one is time dimension. As information carrying signal represents an event, so we can tell that a signal requires four bits of information for its complete specification. So, it is natural to

think that four bits of information can be transmitted by a signal. Next we shall describe how it is possible.

Suppose a three dimensional picture is transmitted, which carries the bit values. As a naive method of transmission, we can think that this three dimensional signal is projected on a screen of the receiver, who by measuring /observing the picture can recover the bit values. Suppose δx_i , δy_i and δz_i are length, breadth and height of the picture. Now δx_i can take any of the two values δx_0 and δx_1 which represent bit 0 and 1 respectively. Similarly δy_i can be δy_0 or δy_1 and δz_i can be δz_0 or δz_1 . So each figure contains three bits simultaneously. The probable sets of three bits are: (0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0) and (1,1,1). These triple bits are generated due to three dimensional space. We can call these bits as space-bits. Let us see how time dimension can be incorporated to generate another bit from the same picture. Suppose sender transmits the picture over the time period δt_i , where δt_i can take any two different values δt_0 and δt_1 . Let us call them time bits. The two values of time duration can represent bit 0 and 1 respectively. So each figure contains 4 bits of which three are space bits and one is time bit. Therefore information can be encoded into the 16 possible sets of 4 bits. The probable sets are: (0,0,0,0), (0,0,0,1), (0,0,1,0), (0,0,1,1), (0,1,0,0), (0,1,0,1), (0,1,1,0), (0,1,1,1), (1,0,0,0), (1,0,0,1), (1,0,1,0), (1,0,1,1), (1,1,0,0), (1,1,0,1), (1,1,1,0), (1,1,1,1). These bits can be called as space-time bits.

Is it possible to transmit four spec-time bits by the existing information processing technique ? In the existing technique, an electromagnetic/voltage pulse carries the bit value. In three dimensional configuration, voltage pulse v can be expressed as $v = (v_x i + v_y j + v_z k)^{1/2}$ where i , j and k are three unit vectors along x , y and z axis. To recover the bit values receiver measures the magnitude $(v_x^2 + v_y^2 + v_z^2)^{1/2}$ of the voltage pulse. Note that he measures the

resultant of three components of voltage pulse as they have no independent existence. Therefore receiver can extract only one bit of information by measuring the voltage of the voltage pulse. It means sender can dump only one bit of information into three dimensional voltage pulse. But time bit can be sent along with the voltage pulse. Let voltage v_0 and v_1 represent two bit values. Suppose the time width of the pulse can be either δt_0 and δt_1 respectively 0 and 1. Therefore, receiver can recover two pulses by measuring the voltage and time duration of the pulse provided $v_0 \neq 0$ and $v_1 \neq 1$. The probable sets of two bits are $:(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$. In this way, two bits can be transmitted by one voltage pulse.

The presented space-time encoding is perhaps a classical encoding since space-time has no quantum analogue. Over the last few years the quantum information has been a major area of research [3, 4]. In quantum information, quantum state is the carrier of information where a single quantum state or a sequence of quantum states [3,4] can represent a bit value. So the same question can be put: Is it possible to realize our space-time encoding in quantum information ? Suppose eigen values of spin state are used to represent bit 0 and 1. The state have three eigen values in the x, y and z axis, but they cannot be simultaneously measured. Therefore it is not possible to encode more than one bit by a spin state. In this sense, three dimensional space has no advantage in quantum encoding. So voltage pulse and quantum state are equivalent in this sense. Still they have a difference. We have seen one voltage pulse can transmit 2 bits. But one quantum state cannot transmit more than one bit. The reason is simple. In quantum encoding we can use either two orthogonal states or two nonorthogonal states to represent two bit values. But nonorthogonal states cannot generate bit for each state (the probability of recovery of a bit value is less than one), only orthogonal states can do so, . So we are interested to know whether two bits per quantum state can be generated by

two orthogonal states. For clarity, suppose $|\uparrow\rangle$ and $|\leftrightarrow\rangle$ are two orthogonal polarization states, which represent 0 and 1. Receiver uses 0° or 90° analyzer to recover bit values. Therefore, receiver could recover 100% transmitted bits if time of transmission is a priori known. Note that 50% bit will be recovered completely from null results. These 50% null results cannot provide more than 50% bit values. It means 2 bits per quantum state cannot be generated. Four dimensional space-time has no advantage in quantum encoding.

Throughout the development of classical information theory, many authors tried to establish a relation between thermodynamics and information theory. In particular, they tried to know the minimal energy cost of information processing. von Neumann [3] and Brillouin [4] argued that $KT \log 2$ energy is required to process a bit. Gabor argued [5] transmission of one bit of information by electromagnetic waves of radio frequency requires a minimum energy KT . On the other hand Shannon formula of channel capacity [1] states that average energy requirement of one bit transmission is $KT \log 2$. Gabor's and Shannon's results are not identical because Shannon formula is not applicable to the transmission of one-bit of information. At present it is widely believed that $KT \log 2$ is required to transmit a bit. But from the earlier work it is not clear whether this energy is dissipative in nature or not. On the question of dissipation, Landauer argued [6] that $KT \log 2$ would dissipate whenever a bit will be erased, otherwise not. Erasure of information is frequently done in computation. Following Landauer's argument Bennett argued [7] that *computation* can be done without spending any energy when computation does not require any erasure of information. This new model of computation is known as reversible computation. From these works one may think dissipation-less information processing is possible only when it is processed reversibly. In a separate work Landauer proposed an alternative encoding [8]

and argued that one bit of information could be transmitted without any energy dissipation. It means reversibility is not a necessary condition for dissipation-less information processing. But, according to Landauer's statement [9] his work created little attention. In true sense, Landauer's scheme is not a bit transmission scheme, rather it is a bit transportation scheme. Therefore one can think $KT\log 2$ result is valid for erasure of bit and transmission of bit.

Many authors [10] cast doubt either on dissipation-less computation or on the issue of energy requirement of erasing a bit because of the lack of mathematical proof. Apart from the lack of proof, $KT\log 2$ result is based on the assumption that a signal contains only one bit of information. We have seen a signal can contain four bits of information. The transmission of this signal or erasing the signal requires $KT\log 2$ energy (assuming the result is valid). The energy cost of transmission of four bits or erasing the four bits is $KT\log 2$. It means encoding cannot be overlooked to determine the energy requirement of bit processing. It also implies space-time will have a role on the fundamental limit's of energy cost of classical information processing. But for quantum information, space-time will not have such similar role because it is not possible to send more than one bit by a quantum state.

In conclusion, information bearing capacity of space-time has been demonstrated. In this encoding space and time have been used on equal footing. On the other hand this encoding qualitatively supports the notion of four dimensional space-time. We hope connection between space-time and information theory might be an interesting topics of research.

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